Probability Density Function

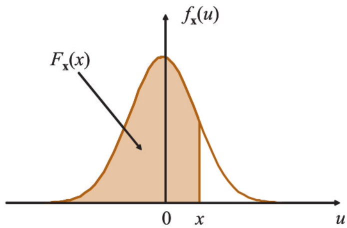
If $\mathbf{x}$ is a **continuous RV**, its **probability density function (PDF)** is related to its CDF by

$$f_{\mathbf{x}}(x) = \frac{dF_{\mathbf{x}}(x)}{dx}$$

Thus, the CDF can also be recovered from the PDF via integration, i.e.,

$$F_{\mathbf{x}}(x) = \int_{-\infty}^{x} f_{\mathbf{x}}(u)du$$

The shaded area in the figure represents the CDF; hence,



$$\Pr(a < \mathbf{x} \le b) = F_{\mathbf{x}}(b) - F_{\mathbf{x}}(a) = \int_{a}^{b} f_{\mathbf{x}}(u)du$$

Because the probability $F_{\mathbf{x}}(x)$ is nondecreasing, it follows that

$$f_{\mathbf{x}}(x) \ge 0, \quad -\infty < x < \infty$$

Also, by virtue of axiom 2, we see that

$$\int_{-\infty}^{\infty} f_{\mathbf{x}}(x)dx = 1$$

That is, the total area under the PDF curve is always unity.

For a **discrete RV** $\mathbf{x}$ that takes on values x_k with probabilities $\Pr(\mathbf{x} = x_k)$, $k = 1, 2, 3, \ldots$, it follows that

$$F_{\mathbf{x}}(x) = \sum_{k=1}^{\infty} \Pr(\mathbf{x} = x_k) U(x - x_k), \quad f_{\mathbf{x}}(x) = \sum_{k=1}^{\infty} \Pr(\mathbf{x} = x_k) \delta(x - x_k)$$

where $U(x - a)$ is the unit step function, and $\delta(x - a) = dU(x - a)/dx$ is the Dirac delta function.